Lecture 8 - Beyond the Standard Model

The Standard Model is amazing. It is the best tested theory ever constructed by humans and accounts for physics from macroscopic scales (more honestly, say starting w/ $E \sim 10^{-2} m_e$ or so when relativity and quantum mechanics both start to matter) to the scales probed by the LHC $E \sim 10$ TeV (we also have high energy cosmic rays and other astrophysical probes, so this is also something of an understatement).

Nonetheless, there are a number of observational phenomena that can not be explained by the SM, and a few theoretical puzzles too.

Observational

- neutrino mass
- dark matter
- baryon asymmetry
- inflation
- dark energy

Theoretical

- Electroweak hierarchy problem
- Cosmological constant problem
- Strong CP problem
- Grand unification?
- Quantum gravity

Neutrino mass

Remember that we always think of the SM as an EFT. This tells us we should include all gauge + Lorentz invariant irrelevant operators supressed by a heavy scale Λ and always keep in mind that our observables are an expansion in E/Λ. The only type of operators we can write at dimension 5 are

$$\mathcal{L} \supset \frac{1}{\Lambda}(HL^{c\dagger})(HL)$$

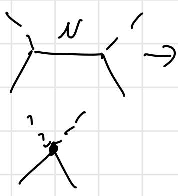
where L is a Weyl spinor lepton doublet $L = \binom{\nu}{e}$ and "c" is charge conjugation.

(Note in this way of writing it both H & L are in $1/2$ rep of $SU(2)_L$ so they are contracted with $\varepsilon^{\alpha\beta}$.) When $\langle H\rangle = \binom{0}{v}$, this gives the neutrinos a "Majorana mass"

$$\mathcal{L} = m_\nu \nu_L^{c\dagger}\nu_L, \qquad w/ \quad m_\nu = \frac{v^2}{\Lambda}$$

What sets the scale Λ? We can generate it by integrating out a heavy new fermionic state N, which is a singlet under all the SM gauge groups.

$$\mathcal{L} = \frac{1}{2}MN^{c\dagger}N + yHL^{c\dagger}N + h.c.$$



$$\Rightarrow \mathcal{L}_{EFT} = \frac{y^2}{M}(HL^{c\dagger})(HL)$$

We can just as easily calculate the mass M assuming $y \sim O(1)$ and knowing $m_\nu \sim 10^{-2}\,eV$ from data $\Rightarrow M \sim$ few $\times 10^{15}$ GeV $< M_{Pl} \sim 10^{18}$ GeV.

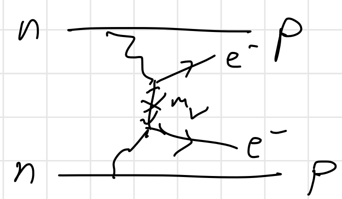
The scaling $m_\nu \sim \frac{v^2}{M}$ is called the seesaw mechanism, since making M larger causes m_ν to be smaller.

- There is another option, which is to add the right handed neutrinos to the SM at low energies and to simply introduce a Dirac mass: $\mathcal{L} = y H L^\dagger N + h.c. \Rightarrow \mathcal{L} = m_D L^\dagger N + h.c.$

In this case, the yukawa coupling would need to be unnaturally small.

Note Majorana neutrinos break lepton number while Dirac neutrinos do not.

$\Rightarrow$ ν-less double β decay



Once neutrinos have mass $\Rightarrow$ CKM-like structure appears called PMNS matrix.

The Hierarchy Problem

The only two relevant parameters in the SM are the vacuum energy (cosmological constant) and the Higgs mass squared parameter. These parameters are not protected by any symmetry in the SM. The naive (Wilsonian or 't Hooft naturalness) expectation is that they would be proportional to the largest physical scale in the theory, e.g. $M_{Pl} \sim 10^{18}$ GeV. Observationally,

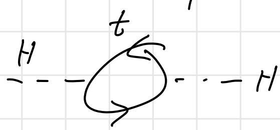
$$\rho_V \simeq 10^{-45}\ \text{GeV}^4 \quad \text{vs.} \quad (M_{Pl})^4 \sim 10^{72}\ \text{GeV}^4$$

- Let's focus on the Higgs mass parameter:

  $m_H^2 \simeq (88\,\text{GeV})^2$ vs. M_{Pl} is also a big problem.

  We can make this more precise. The typical argument given is that the sensitivity to high energy scales is due to a loop correction involving the largest coupling involving the Higgs $\Rightarrow$ the top Yukawa

People often say the hierarchy problem is the statement that



$$\sim \frac{y_t^2}{16\pi^2}\Lambda_{UV}^2$$

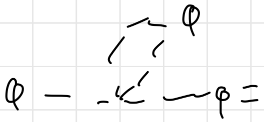
However, this is imprecise and can lead to confusion (or people saying things that are simply wrong).

We make a distinction between <u>calculable</u> and <u>incalculable</u> contributions to low energy parameters. The quadratic divergence itself is a manifestation of the fact that we are working with an EFT. It is an incalculable UV dependent contribution to the fundamental parameter. However, if we introduce a more fundamental model (with a new heavy scale M), we can see that the mass scale will appear as a quadratic correction to the mass parameter.

We can sharpen this further using a toy model. Note that "there are no quadratic divergences in dim reg." In particular, take a model of two scalars φ (mass m) + Φ (mass M) w/

$$\mathcal{L}_{int} = -\frac{\lambda}{4}\varphi^2\Phi^2 - \frac{\eta}{4!}\varphi^4$$

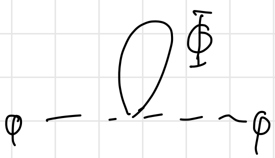
(see TASI lectures arXiv:1903.03622)



$$= \frac{i\eta}{32\pi^2} m^2\left[\frac{1}{\varepsilon} + \log\frac{\mu^2}{m^2} + 1 + \mathcal{O}(\varepsilon)\right] \quad (\overline{MS}\ \mu)$$

looks like no "quadratic divergence".

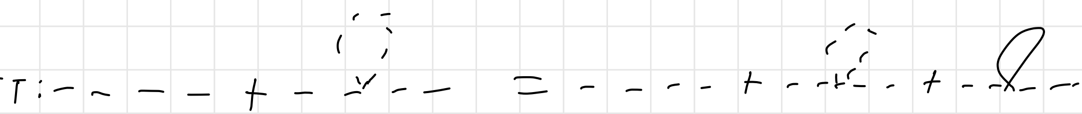
There is another loop in the full theory:



$$= \frac{i\lambda}{32\pi^2} M^2\left[\frac{1}{\varepsilon} + \log\frac{\mu^2}{M^2} + 1 + \mathcal{O}(\varepsilon)\right]$$

We can also study the EFT that we get from integrating out Φ. If we "match" the propagators between the UV theory + EFT, we find

EFT:



UV theory

$$\Rightarrow \quad m^2_{EFT}(\mu = m_{EFT}) - \frac{\eta}{32\pi^2} m^2_{EFT}\left(1 + \log\frac{M^2}{m^2_{EFT}}\right)$$

$$= m^2_{UV}(\mu = M) - \frac{\lambda}{32\pi^2} M^2$$

If we want $m^2_{EFT} \ll M^2$, we have to fine tune m^2_{UV} against M^2. This is the hierarchy problem.

- How can we solve this problem? We need to enlarge the symmetry so that the scalar mass squared becomes a spurion. Then in the limit that $m^2_{\text{breaking}} \to 0$ there is an enlarged symmetry $\Rightarrow$ no quadratic contributions from crossing heavy mass thresholds. You already know one example of this: if φ is a pseudo Goldstone, then its mass is a spurion of shift symmetry breaking. The second known way is to introduce <u>supersymmetry</u>, a symmetry between bosons and fermions.

(see Martin's SUSY Primer arXiv: hep-ph/9709356)

Take a Theory of a free massless Scalar φ and a free massless Weyl fermion ψ. We want to transform the scalar into the fermion $\Rightarrow$ introduce a Grassmann parameter ε: $\quad \delta\varphi = \varepsilon\psi$

Then The Scalar kinetic term $\mathcal{L}_\varphi = -\partial^\mu\varphi^* \partial_\mu\varphi$ transforms at $\mathcal{O}(\varepsilon)$ into $\delta\mathcal{L}_\varphi = -\varepsilon\partial^\mu\psi\partial_\mu\varphi^* + \text{h.c.}$

We need a fermionic transformation That cancels

$$\delta\mathcal{L}_\varphi \Rightarrow \delta\psi_\alpha = -i(\sigma^\mu\varepsilon^\dagger)_\alpha \partial_\mu\varphi$$

Then $\mathcal{L}_\psi = i\psi^\dagger\bar{\sigma}^\mu\partial_\mu\psi \Rightarrow \delta\mathcal{L}_\psi = -\varepsilon\sigma^\mu\bar{\sigma}^\nu\partial_\nu\psi\partial_\mu\varphi^* + \text{h.c.}$

Using $[\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu] = -2\eta^{\mu\nu}$

$$\Rightarrow \delta\mathcal{L}_\psi = \varepsilon\partial^\mu\psi\partial_\mu\varphi^* + \varepsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\varphi$$
$$- \partial_\mu\left(\varepsilon\sigma^\nu\bar{\sigma}^\mu\partial_\nu\varphi^* + \varepsilon\psi\partial^\mu\varphi^* + \varepsilon^\dagger\psi^\dagger\partial^\mu\varphi\right)$$

$$\Rightarrow \delta S = \int d^4x \left(\delta\mathcal{L}_\varphi + \delta\mathcal{L}_\psi\right) = 0$$

One can further show that for two SUSY transforms δ_1 + δ_2: $(\delta_2\delta_1 - \delta_1\delta_2)\varphi = i(-\varepsilon_1\sigma^\mu\varepsilon_2 + \varepsilon_2\sigma^\mu\varepsilon_1)\partial_\mu\varphi$

$\Rightarrow$ SUSY transform is like "square root of derivative" $\Rightarrow$ extension of spacetime symmetry. Can be extended to interacting theories!

- How does SUSY solve the hierarchy problem? The scalar and fermion must have the same mass $\Rightarrow$ when their mass $\to 0$ chiral symmetry is restored $\Rightarrow$ scalar mass acts as spurion of chiral symmetry breaking. (Loop corrections to electron mass $\to 0$ as $m_e \to 0$)

Strong CP problem and the Axion

This is an advanced topic. We will only hit the highlights (see Hook TASI arXiv:1812.02669) (There is also some nice related material in Srednichi book) (Also Donoghue et al book)

There is an important quantum effect known as the "anomaly". It is a quantum breaking of a classical symmetry. One famous example of this is that axial rotations of fermions are anomalous $\Rightarrow$ if I rotate the up quark by $u \to e^{i\alpha} u$ and $u^c \to e^{-i\alpha} u^c$

$$\Rightarrow \mathcal{L} \to \mathcal{L} + \alpha \frac{g^2}{16\pi^2} G\tilde{G}$$

Imagine my fundamental $\mathcal{L}$ had a term

$$\mathcal{L} \supset \theta \frac{g^2}{32\pi^2} G\tilde{G} \Rightarrow \text{rotation by } \alpha \text{ shifts } \theta \to \theta - 2\alpha.$$

Let us include this effect into the theory of pions with only u + d. Then the transformation rule becomes $u \to e^{i\alpha} u \quad d \to e^{i\alpha} d \quad \theta \to \theta - 2\alpha$

In the chiral $\mathcal{L}$ we have

$$\mathcal{L} = f_\pi^2 \, \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \, \mathrm{Tr}\, M U + \text{h.c.}$$

where $U = e^{i\pi^a \sigma^a/\sqrt{2} f_\pi}$ w/ $\sigma^0 = \mathbb{1}$

σ^a = Pauli matrices

and $\pi^0 = \eta' \qquad \pi^3 = \pi^0$

$\pi^{1,2} \Rightarrow \pi^{+/-}$

Note $U \to e^{i\alpha} U$, $\theta \to \theta - 2\alpha$, $M \to e^{-i\alpha} M$

$\Rightarrow \eta' \to \eta' + \sqrt{2}\alpha f_{\eta'}$

Since the axial symmetry is broken, we can also write $\mathcal{L} \supset b f_\pi^4 \, \mathrm{Tr}\, U$

If we write $b = |b| e^{i\theta}$. Allowing a phase in the quark mass matrix too, $M = \begin{pmatrix} m_u e^{i\theta_u} & \\ & m_d e^{i\theta_d} \end{pmatrix}$

One can compute the pion masses in the presence of these angles: (see Hook notes)

$$m_{\pi^0}^2 = a f_\pi \sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos\bar{\theta}} \,, \quad m_{\pi^{+/-}} = a f_\pi (m_u + m_d)$$

w/ $\bar{\theta} = \theta + \theta_u + \theta_d$. Note $m_{\pi^0} \simeq m_{\pi^+} \Rightarrow \bar{\theta}$ should be small

Now we can introduce the nucleons:

$N = \begin{pmatrix} p \\ n \end{pmatrix}$ transforms in an $SU(2)$ doublet.

The leading order Lagrangian is then

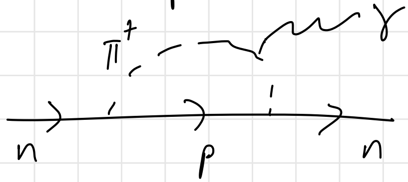
$$\mathcal{L} = -m_N N U^\dagger N^c - c_1 N M N^c - c_2 N U^\dagger M^\dagger U^\dagger N^c$$
$$- \frac{i}{2}(g_A - 1)\left[N^\dagger \bar{\sigma}^\mu U \partial_\mu U^\dagger N + N^{c\dagger} \bar{\sigma}^\mu U^\dagger \partial_\mu U N^c \right]$$

Expanding this out gives

$$\mathcal{L} \supset -\bar{\theta} \frac{c_+ \mu}{f_\pi} \pi^a N \tau^a N^c - i \frac{g_A m_N}{f_\pi} \pi^a N \tau^a N^c$$

w/ $\mu = \dfrac{m_u m_d}{m_u + m_d}$ and $c_+ = c_1 + c_2 \simeq 1.7$ & $g_A \simeq 1.27$ } measured

The $\bar{\theta}$ dependent term $\Rightarrow$ neutron EDM:



$4\pi f_\pi$

$$\Rightarrow \quad d_n = \frac{e\bar{\theta} g_A c_+ \mu}{8\pi^2 f_\pi^2} \log \frac{\Lambda^2}{m_\pi^2}$$

$$\simeq 3 \times 10^{-16}\, \bar{\theta}\ e\,\text{cm}$$

for $\mathcal{L} \supset d_n F_{\mu\nu} \bar{n} \gamma^{\mu\nu} i \gamma^5 n$

Comparing to bound on neutron EDM $\Rightarrow \bar{\theta} \lesssim 10^{-10}$

- What does θ mean since it is coefficient of a total derivative? $\mathcal{L} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \frac{\theta g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$

It turns out there are pure gauge field configurations that exist at the boundary of spacetime and cannot be continuously deformed into each other. They wrap around the sphere at infinity and are charictorized by a "winding number".

$$\int d^4x \frac{1}{32\pi^2} G\tilde{G} = n_1 - n_2$$

One can connect these different sectors using "instantons"

The physical gauge invariant vacuum is actually $|\theta\rangle = \underset{\uparrow}{N} \sum' e^{i\theta n} \underset{\uparrow}{|n\rangle}$

norm — winding number

This θ is the parameter in front of $G\tilde{G}$ since for a generic gauge invariant operator $\mathcal{O}$:

$$\langle\theta|\mathcal{O}|\theta\rangle = \sum_{m,n}' e^{i\theta(m-n)} \langle m|\mathcal{O}|n\rangle$$

$$= \sum_{\Delta,n}' e^{i\theta\Delta} \langle n+\Delta|\mathcal{O}|n\rangle = \sum_{\Delta,n}' e^{i\theta\int d^4x \frac{1}{32\pi^2} G\tilde{G}} \langle n+\Delta|\mathcal{O}|n\rangle$$

$$= \sum_{\Delta} \int \mathcal{D}A \, e^{i\int d^4x \left(\mathcal{L} + \frac{\theta}{32\pi^2} G\tilde{G}\right)} \delta\left(\Delta - \int d^4x \frac{1}{32\pi^2} G\tilde{G}\right) \mathcal{O}$$

$$= \int \mathcal{D}A \, e^{i\int d^4x \left(\mathcal{L} + \frac{\theta}{32\pi^2} G\tilde{G}\right)} \mathcal{O}$$

- The Axion idea is to promote $\bar{\theta}$ to a dynamical field. Let a be a pseudo scalar. We want UV models that generate a coupling

$$\mathcal{L} \supset \left(\frac{a}{f_a} + \theta\right) \frac{1}{32\pi^2} G\tilde{G}$$

so that under the anomalous symmetry $\theta \to \theta - \alpha$, $a \to a + \alpha f_a$

One can compute the potential for the chiral Lagrangian including the axion:

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{m_u + m_d} \sin^2\left(\frac{a}{2f_a} + \frac{\bar\theta}{2}\right)}$$

$\Rightarrow$ the min of the potential is $\langle a \rangle = -\bar\theta f_a$

$\Rightarrow d_n \sim \bar\theta + \frac{a}{f_a} = 0$

This happens if the axion is a Goldstone boson of a theory with an anomalous Global $U(1)_{PQ}$ (PQ = Pecci - Quinn). If Φ is a heavy scalar charged under $U(1)_{PQ}$ and

$$V = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + y \Phi q q^c$$

new heavy quarks

$\Rightarrow \langle \Phi \rangle = f_a$ and $\Phi = (f_a + s)\, e^{ia/f_a}$

Then a chiral rotation of these heavy quarks shifts the axion into coeff of $G\tilde{G}$. The coupling $\frac{a}{f} G\tilde{G}$ (and can also have other couplings) leads to tons of pheno consequences, e.g. the axion could be dark matter.